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Laboratory studies on magnetic water treatment and their relationship to a possible mechanism for scale reduction

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Abstract

Laboratory studies indicate that magnetohydrodynamic effects may be responsible for claims that magnetic treatment devices are sometimes effective for scale control in water-using systems. In addition to enhancing corrosion of metals in the vicinity of the device, or within the device itself, application of the field 90° to the flow of a conducting fluid can alter the hydrodynamics of fluid flow. Depending on experimental conditions, this may increase or decrease turbulence in the fluid, promoting aggregation or deaggregation of both ferromagnetic and diamagnetic colloids. Important factors in promoting magnetohydrodynamic forces on fluid flow include conductivity of the solution, linear flow velocity of the fluid, and the flux density (magnetic induction) of the transverse field. Finally, magnetic treatment devices that are physically designed to create additional turbulence by constricting or otherwise altering fluid flow may further enhance the anti-scaling effect by purely mechanical means.

Keywords: Magnetohydrodynamics; Magnetic water treatment; Scale prevention

1. Introduction

Non-chemical water treatment devices were first proposed as a means of scale control in 1865 [1,2]. In 1873, A.T. Hay received the first US patent for a water treatment device that employed a magnetic field [3]. Today, many of these devices are commercially available. Some employ one magnet, some two or more. In some, the magnet is located inside the pipe through which the treated water flows; in others the magnet is placed outside of the pipe. Although the variety of devices on the market may seem nearly infinite, most can be classified into four basic types [4].

Whatever the design, even a cursory review of the literature surrounding these devices reveals numerous contradictions in claimed effects, low

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reproducibility of results, and explanations based on strange and mysterious effects of the field on the structure of water and/or interactions of the field with dissolved minerals that are generally diamagnetic [5,6]. In the West, the literature surrounding magnetic water treatment has been almost uniformly negative [4,7–11], although there are a limited number of successful applications that appear quite credible [12–14]. Among the positive reports originating in the West, work by the well-known scientist, Marcel Pourbaix, is especially notable [15–17]. Thus, in spite of many decades of research and testing, the major problems in accepting claims made for magnetic treatment devices (MTDs) remain (1) the lack of a fundamental, scientifically acceptable explanation connecting magnetic fields to the precipitation of scale-forming minerals, and (2) the paucity of readily reproducible data defining the conditions under which the treatment is most effective, or in fact, under which the treatment will work at all.

This paper is an outgrowth of a lecture presented at a symposium on magnetic water treatment held at Cranfield University during the spring of 1996 [18]. This lecture did not attempt a comprehensive review of the massive literature in the area. Rather, it attempted to summarize the work conducted in our own laboratory over the past 14 years.

Our own search for possible mechanisms that might account for the claims made for MTDs began in 1982 as a result of field studies conducted by Amoco Oil Company [12,13]. In 1982, Grutsch and McClintock [12] reported that a magnetic treatment device was employed successfully on a cooling tower at their Texas City refinery. Following MTD installation, the tower was operated at concentrations of up to 35 cycles for several months without chemical treatment. This apparently successful application of MTD technology led to our first study at Baylor University, which was funded by the American Petroleum Institute and later published [5] as API Publication 960. The stated goal of our study was to determine if there was *any* scientific basis for the observed scale-reduction.

At roughly this same time, Szostak and Toy [14] reported another apparently successful application of MTD technology on a cooling tower at Big Three Industries, a leading manufacturer of compressed and liquified atmospheric gases located in Bayport, Texas. In this application, a cooling tower with a volume of 70,000 gal and a recirculation rate of 5000 gpm was operated for 18 months without chemical treatment. During this period, it was claimed that the cooling tower water was concentrated in excess of 50 cycles, and a fine sludge of soft material accumulated in the tower basin to a depth of about 3".

Since the initial API report [5] in 1985, our laboratory has continued to investigate the effect of magnetic fields on both scaling and non-scaling materials. While many of these results have already appeared in peer-reviewed journals, their relationship to magnetic water treatment may not be readily apparent. In this paper, we will attempt to summarize our results and relate the significance of our laboratory studies to an understanding of why magnetic water treatment might affect the deposition of scale-forming minerals in certain commercial water-using systems. The work can be divided into three phases: Phase I involves fundamental studies of the physical and chemical effects observed during the operation of one type of commercially available MTD; Phase II involves calcium carbonate scaling experiments using a bank of six, simultaneously operating stills; and Phase III involves a study of the effect of externally applied magnetic fields on diamagnetic and weakly ferromagnetic colloidal suspensions.

2. Phase I—fundamental studies using a commercial MTD

Our earliest studies [5,19–22] employed MTDs similar to the one used by Grutsch and McClintock [12,13]. Our goal, initially, was

simply to determine what effect, if any, resulted or was induced when water passed through a commercial MTD according to manufacturer's specifications.

2.1. The test rig

The test rig [5,21] consisted of a circulation loop constructed of PVC pipe, a variable speed pump, cooling coils to maintain a constant temperature in the circulating fluid, and ports to admit a variety of measuring devices. Because most successful US applications [12–14] had employed MTDs in which the magnetic field was oriented at 90° to the direction of fluid flow, all of our laboratory studies have employed a similar orientation. The Phase I rig produced a linear fluid flow velocity of about 1.5 m s^{-1} through the transverse field, as specified by the manufacturer.

2.2. Magnetic treatment devices

Fig. 1 shows a schematic diagram of the commercial MTD used in Phase I studies. The device consisted of a spool piece (5-cm diameter) with two magnetic rods symmetrically oriented along the axis. Pole pieces at the end of each permanent magnet focused the magnetic field perpendicular to the fluid flow and accelerated the linear velocity of the fluid flow in the vicinity of the magnetic field.

For test purposes, the manufacturer supplied four different types of MTDs (Table 1): a magnetized unit with metallic housing (unit 1), a magnetized unit with PVC housing (unit 2), an unmagnetized unit with metallic housing (unit 3) and an unmagnetized unit with PVC housing (unit 4). The magnetized units had a measured induction of 1250 and 1500 G (0.125 and 0.150 T) at the two opposite ends of the unit as measured in the gap between the pole pieces. The unmagnetized MTDs, used for control experiments, were identical to the magnetized units in every way except that the rods had not been magnetized at the factory. These unmagnetized

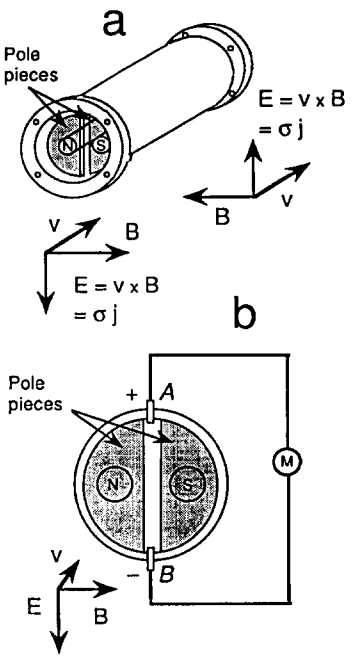

Fig. 1. (**a**) Schematic diagram of the magnetic treatment device used in Phase I studies. Vector diagrams show the orientation of the fluid flow velocity vector v, magnetic induction vector B, and electric field and current vectors E and j, respectively, for the two magnetic fields within the device, and σ denotes solution conductivity. Note that the downstream magnetic field formed by the downsteam ends of the magnetic rods (not shown) is opposite to the upstream field direction. (**b**) Orientation of electrodes used in electrical measurements. *A*, positive electrode; *B*, negative electrode; M, voltmeter; N, magnet north pole; S, magnet south pole. Pole pieces serve to focus the magnetic field and increase the solution flow rate in the gap.

units had a measured induction of 20 G (0.0020 T) in the gap between the pole pieces. Unit 5, a piece of PVC pipe with no rods or other obstructions, was used for additional control experiments.

2.3. Electrical measurements

Using the treatment rig and MTDs described above, several types of electrical measurements were made [19–22]. From basic magnetohydrodynamics [23–26], it is known that a potential will be generated whenever a conducting

Table 1
Construction features for units used in Phases I and II

Unit	Housing material	Magnet assembly[a]		Magnetized	
		Yes	No	Yes	No
1	Metallic	X		X	
2	Metallic	X			X
3	PVC	X		X	
4	PVC	X			X
5	PVC		X		X

[a]Magnet assembly consists of pole pieces and associated ferromagnetic rods. Rods may be either magnetized or not as indicated above.

fluid (i.e., a dilute electrolyte) flows through a transverse magnetic field. This is the principle behind the well-known magnetic flow meter [24]. According to theory, under ideal conditions the voltage generated by the flow of a conducting fluid will be given by

$$e_T = e_S + \left[\left(\vec{V}\times\vec{B}\right)L\right] \quad (1)$$

where e_T is the measured voltage, e_S is any potential difference that may exist in the absence of fluid flow, $\vec{V}$ is the linear fluid velocity vector, $\vec{B}$ is the magnetic induction vector, and L is the distance between the electrodes. Because both $\vec{V}$ and $\vec{B}$ are vector quantities, the term $\vec{V}\times\vec{B}$ represents the vector cross product of $\vec{V}$ and $\vec{B}$ (Fig. 1). Thus, the magnitude of the induced voltage, $\left(\vec{V}\times\vec{B}\right)L$, will be greatest when the direction of the applied magnetic field is orthogonal to the direction of fluid flow. Eq. (1) assumes that the Hall effect can be neglected, which is normally true for aqueous solutions [24].

While the passage of water containing dissolved ions through an MTD should theoretically produce a voltage according to Eq. (1), this had never been demonstrated for a commercial, off-the-shelf MTD operating under manufacturer's specifications. To make these voltage measurements, pairs of electrodes were installed 180° apart in the PVC-walled MTD (unit 3, Table 1) as shown in Fig. 1b.

Fig. 2 shows the voltages obtained as a function of solution flow rate for three different electrode materials (brass, stainless steel, and Ag/AgCl). As predicted from Eq. (1), a plot of measured voltage versus linear flow rate is a straight line. Curve 4 in Fig. 2 was made using an unmagnetized MTD and demonstrates that no voltage is produced by flow in the absence of the magnetic field.

Fig. 3 shows the results of current measurements obtained as a function of solution flow rate using the two stainless steel electrodes. In this figure, curves 1 and 2 correspond to measurements made at 1500 G (0.150 T, curve 1) and 1250 G (0.125 T, curve 2), respectively. Both curves can be interpreted as indicating that an electrochemical reaction begins to occur once a threshold voltage (i.e., flow rate) is reached. Curve 2 requires a somewhat higher flow rate for the reaction because the magnetic induction was less at this end of the unit.

2.4. pH measurements

If electrochemical reactions can be induced as a result of MTD operation (Fig. 3), then such reactions might be pH-dependent or might alter the pH of the solution as they occurred. Our next experiments were designed to determine the effect of MTD operation on solution pH [20,21]. In these experiments, both bulk and surface pH

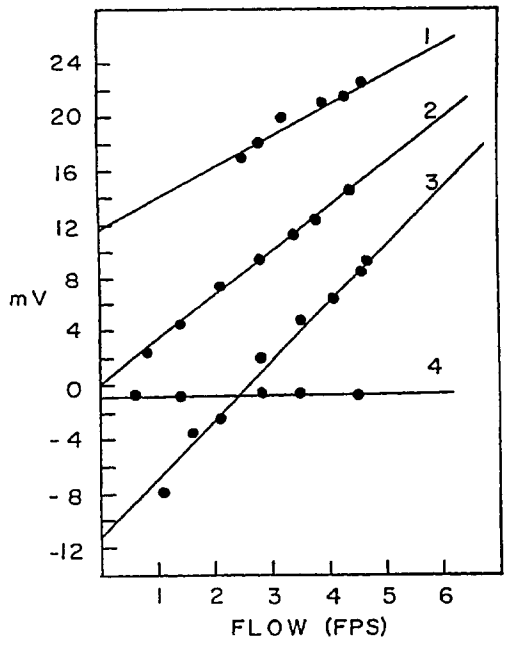

Fig. 2. Measured voltage versus solution flow rate. Curve 1, brass electrodes; Curve 2, Ag/AgCl electrodes; Curve 3, stainless steel electrodes; Curve 4, Ag/AgCl electrodes; solution conductivity 2,100 $\mu\Omega^{-1}$ (0.019 M chloride); Curves 1, 2, and 3 using unit 3 MTD (Table 1) in 0.125 T portion of the magnetic field; Curve 4 using unit 4 MTD with 0.0020 T magnetic field. Flow rate through the magnetic field is approximately four times the value indicated in the figure, due to the presence of the pole pieces. (Reprinted from [19] and [21].)

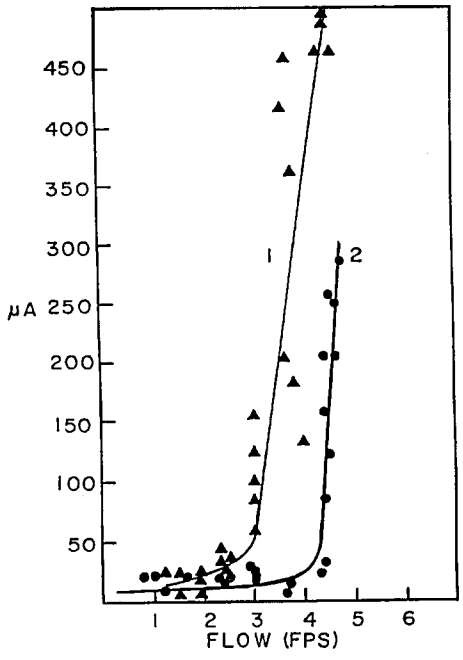

Fig. 3. Measured current versus solution flow rate. Unit 3 MTD (Table 1); Curve 1, 0.150 T portion of the magnetic field; Curve 2, 0.125 T portion of the magnetic field; solution conductivity 2,100 $\mu\Omega^{-1}$ (0.019 M chloride); stainless steel electrodes. Flow rate through the magnetic field is approximately four times the value indicated in the figure, due to the presence of the pole pieces. (Reprinted from [19] and [21].)

values were monitored simultaneously. Bulk solution pH was determined using a conventional combination glass/reference electrode inserted into the circulating solution, while surface pH measurements were made by positioning a micro-glass pH electrode close to the surface of the MTD.

Fig. 4 shows a plot of surface pH minus bulk solution pH as a function of time for an unmagnetized MTD (unit 2, Table 1). The two curves correspond to measurements made at the top and bottom of the gaps between the pole pieces (Fig. 1b). For comparison, the difference between surface pH and bulk solution pH for a section of PVC pipe (unit 5, Table 1) is also shown. Fig. 4 indicates that both surfaces of the

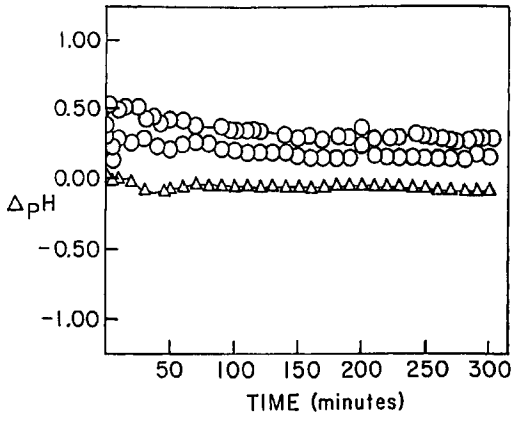

Fig. 4. Surface pH minus bulk pH (ΔpH) vs recirculation time for both surfaces of unit 2 (unmagnetized MTD, Table 1) compared to PVC pipe. ○, unit 2 (unmagnetized MTD); Δ, unit 5 (PVC pipe). (Reprinted from [21].)

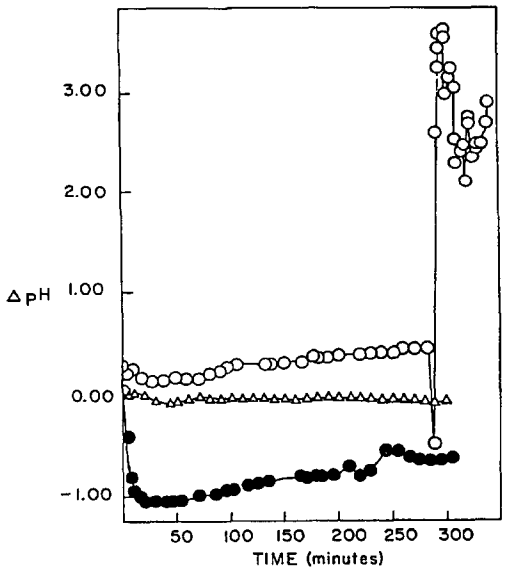

Fig. 5. Surface pH minus bulk pH (ΔpH) vs recirculation time for both surfaces of unit 1 (magnetized MTD, Table 1) compared to PVC pipe. ●,○, unit 1 (magnetized MTD); Δ, unit 5 (PVC pipe). (Reprinted from [21].)

unmagnetized MTD are more alkaline than the bulk solution—a situation typical of ordinary corroding iron where surface pH values as high as 9.5 have been reported [27].

Fig. 5 shows a plot of the difference between surface pH and bulk solution pH as function of time for the magnetized MTD (model 1, Table 1). For comparison, the pH difference for a section of PVC pipe is also presented. It is clear from Fig. 5 that for this commercially available MTD, one surface is more alkaline than the bulk solution, while the other surface is more acidic than the bulk solution.

The pH measurements shown in Figs. 4 and 5 can both be interpreted in terms of simple oxidation-reduction chemistry involving the corrosion of iron. In the unmagnetized MTD, ordinary corrosion of the housing will involve the oxidation of iron,

$$Fe_{(s)} \rightarrow Fe^{2+}_{(aq)} \quad 2e^- \tag{2}$$

and the reduction of either oxygen,

$$\tfrac{1}{2}O_2 + H_2O + 2e^- \rightarrow 2OH^- \tag{3}$$

or water.

$$2H_2O + 2e^- \rightarrow H_2 + 2OH^- \tag{4}$$

Because the anodic and cathodic regions are in close proximity, the OH^- ions released from cathodic regions combine with Fe^{2+} ions formed at nearby anodic regions to produce $Fe(OH)_{2(s)}$.

$$Fe^{2+}_{(aq)} \quad 2OH^-_{(aq)} \rightarrow Fe(OH)_{2(s)} \tag{5}$$

Thus, in the unmagnetized MTD, no matter which reduction half-reaction occurs (Eq. 3 or 4), the pH of the solution at the surface will be alkaline as indicated by the solubility product ($pK_{sp} = 14.5$) of ferrous hydroxide,

$$K_{sp} \quad \left[Fe^{2+}_{(aq)}\right]\left[OH^-_{(aq)}\right]^2 \quad 3.16\times10^{-15} \tag{6}$$

For the magnetized MTD, oxidation and reduction half-cells are physically separated. In the cathodic regions of the MTD housing, the half reaction is represented by either Eq. 3 or 4. Hydroxide ions are produced in either case, and the surface pH should be alkaline relative to the bulk pH.

In the anodic regions of the MTD housing, the oxidation half-reaction represented by Eq. 2 will occur. However, the Fe^{2+} ions will combine with available OH^- ions and the net result,

$$Fe_{(s)} \quad 2H_2O \rightarrow Fe(OH)_{2(s)} \quad 2H^+ \quad 2e^-$$

causes the solution at the surface of the anodic regions to become acidic.

While other half reactions might be used to account for these pH observations, the presence of enhanced corrosion was further supported by an examination of the interior of the magnetized MTD. After a relatively short period of use, the

magnet pole pieces, initially coated with epoxy, became extensively pitted from corrosion. By contrast, the extent of corrosion in the unmagnetized MTD units appeared to be much less.

2.5. Significance of Phase I experiments

Phase I experiments clearly demonstrated that the flow of a conducting fluid through a commercially available MTD produced measurable electrical effects based on well-established scientific principles. The difference in surface pH in two different regions of the housing of the magnetized unit was also consistent with the hypothesis that MTD operation enhances an already spontaneous reaction, the corrosion of iron. To connect these observed effects and claims for scale reduction, we proposed two different hypotheses [5,20,21]. Both involved the generation of nucleation centers on which calcium carbonate would preferentially precipitate (i.e., the formation of a precipitation initiator).

Mechanism I involved conditions at the cathodic regions within the MTD where the presence of high, localized alkalinity near the surface (Eqs. 3 and 4) might produce seed crystals of salts, like $CaCO_3$, that are less soluble at high pH. If the bulk solution was saturated or nearly saturated with such salts, the dissolution rate of these seed crystals might be slow, and they could be carried by fluid flow into regions of bulk supersaturation, where they could initiate rapid, spontaneous precipitation of scale-forming salts in the bulk of the solution.

Mechanism II involved the release of iron corrosion products from anodic regions of the MTD (Eq. 7). A number of studies had reported that the presence of iron is important for the successful operation of an MTD [28], and the possibility that iron-containing materials might serve as nucleation centers for $CaCO_3$ has been suggested by both Russian [29,30] and American investigators [31]. Thus, Mechanism II would involve the adsorption of calcium ions onto the surface of colloidal iron-containing oxides or hydroxides released by the MTD, followed by the heterogeneous precipitation of insoluble calcium salts on the colloidal particles.

Following our initial reports [5,21], Herzog et al. [32] investigated the effect of various iron compounds on calcium carbonate nucleation and growth. These workers reported that none of the seven iron(III) compounds studied—hematite (α-Fe_2O_3), akaganeite (β-FeOOH), goethite (α-FeOOH), lepidocrocite (γ-FeOOH), δ-FeOOH, magnetite (Fe_3O_4), and ferrihydrite ($Fe(OH)_3 \cdot nH_2O$)—were effective as heterogeneous nucleators of calcium carbonate. However, trace amounts of Fe^{2+} were shown to effectively inhibit the growth of calcite (but not aragonite) as well as the transformation of aragonite to calcite. Inhibition of calcite growth by Fe^{2+} was attributed to the precipitation of $FeCO_3$ on the growth sites of the calcite crystal, thereby preventing crystal growth.

While the work of Herzog et al. [32] might seem to rule out our Mechanism II (i.e., heterogeneous nucleation of calcium carbonate by iron corrosion products), it did not rule out Mechanism I (formation of $CaCO_3$ seed crystals in regions of high alkalinity) or the possibility that other colloidal material might act as heterogeneous nucleation centers for scale-forming minerals. Indeed, Herzog et al. [32] did find that tiny crystals of silica acted as heterogeneous nucleation centers for $CaCO_3$ precipitation. Finally, it should be noted that Herzog et al. [32] never studied or even considered how *flowing* their iron colloids through the magnetic field might effect the nucleation properties of their materials.

3. Phase II—statistcal studies using a commercial MTD

By 1986, we had only a hypothetical connection between magnetic fields and scale

inhibition. To test the effect of an MTD on calcium carbonate scaling, we designed a procedure that employed a bank of six, simultaneously operating laboratory stills [33]. Our protocol avoided a number of problems that we believed were associated with many of the negative laboratory tests conducted by other groups: operating conditions which did not match MTD specifications, water conditions which changed throughout the duration of the test, and results which were too few in number for statistical analysis.

Stills were assembled from 1-l glass resin kettles. A copper heating coil was inserted inside each still, and during operation, make-up water from the rig maintained a constant volume of 750 mL in the still. When makeup water was not needed, freshly treated water was returned to the circulation loop. Our new rig design [33] reduced the amount of dead volume to a minimum and insured that only freshly treated water would be fed to each of the six stills.

Three treatment conditions were tested in the protocol by inserting different units in the rig. The commercially available MTD was the same as that used in Phase I (unit 1, Table 1) and produced a magnetic induction of 0.150–0.125 T orthogonal to the direction of fluid flow. For comparison, two controls were employed: an appropriate length of clear PVC pipe (unit 5, Table 1) and the unmagnetized device which was otherwise identical to the commercially available MTD (unit 2, Table 1).

Test waters came from natural sources or were prepared synthetically in the laboratory. Synthetically prepared hard water was made by dissolving calcium chloride and sodium chloride (to adjust conductivity, 1,500 $\mu\Omega^{-1}$) in deionized water, then adding sodium bicarbonate just prior to a run. Natural waters were obtained from the Brazos River, located adjacent to the Baylor campus. Prior to a test, river water was filtered through diatomaceous earth to remove most of the turbidity, then mixed with enough sodium chloride to adjust the conductivity to 1,200 $\mu\Omega^{-1}$.

Using this Phase II rig, fluid flow could be continuously maintained at 152 l min^{-1} through the MTD, as specified by the manufacturer. The total water required to fill the rig for any given test was only about 57 l, and enough test water (760 l) could be prepared for several runs. Thus, variability in water quality could be held constant over a number of runs. During each test, distillation was conducted at a rapid boil until 1,500 mL of distillate was collected. Total distillation time was only about 3 h, and each run produced six data sets. Thus, sufficient data could be collected within a few days for statistical analysis.

Full details of rig construction and the results of these tests are described elsewhere [33]. To summarize, a series of control experiments using PVC pipe was first conducted and subjected to an analysis of variance (ANOVA). The results showed that the data collected for three different runs (18 distillations) were highly reproducible both within any given run and between the three runs. When synthetic solutions were employed, both magnetized and unmagnetized MTD units reduced scale in the stills compared to the PVC control. However, the magnetized unit was somewhat more effective (22% reduction) than the unmagnetized unit (17% reduction). When Brazos River water was employed, the total amount of scale produced in the stills was much greater than that produced by the artificial water, but there was no significant difference in either calcium or magnesium scaling between the magnetized MTD, the unmagnetized MTD or the PVC control.

4. Phase III—Colloid studies using an externally applied magnetic field

MTD applications reported by both Amoco Oil and Big Three Industries [12–14] involved recirculating cooling towers. Another important difference between these two apparently successful MTD applications and typical laboratory studies with MTDs (which are often

negative) is the complexity of the water employed in each test. Laboratory studies, in particular, frequently employ filtered solutions to avoid the presence of colloidal material. We had also filtered our Brazos River water in Phase II experiments to remove much of the turbidity [33]. We concluded that we should next investigate what effect, if any, could be observed when colloidal suspensions were made to *flow* through an orthogonally applied magnetic field. Iron-containing colloids initially seemed a good choice because our Phase I studies [5,20,21] had shown that iron might be released as a by-product of corrosion during MTD operation. To avoid complications from colloidal material generated by MTD operation, we designed a rig that would isolate the circulating fluid from direct contact with all metals, including the magnet.

Colloidal hematite

In the initial stages of Phase III studies [34], hematite sol, α-Fe_2O_3, was chosen as a model colloid because it is well characterized, and samples of highly uniform diameter (monodisperse) are easy to prepare [35]. Hematite is also weakly ferromagnetic and has been the subject of many studies involving the application of magnetic fields under static and quasi-static conditions [34,36–38].

Most theories dealing with the aggregation of colloidal suspensions by applied magnetic fields have simply extended classical DLVO theory [39] by adding a new term to introduce an attractive magnetic interaction between the particles and the applied field [38,40]. Above some threshold value [38,40] of the magnetic field, this magnetic interaction term promotes colloidal instability and consequent flocculation (i.e., the formation of loose aggregates) in a secondary potential minimum. According to this theory, the tendency to aggregate should be proportional to the second power of the applied field strength, the second power of the magnetic susceptibility of the solid phase (i.e., the colloidal particles), and the sixth power of the particle radius. Using this approach, a minimum particle diameter of about 1 micron should be required [38,40] for a magnetic induction of 0.1 T to have a significant aggregating effect on α-Fe_2O_3.

Following standard procedures [35], we prepared a nearly monodisperse sample of α-Fe_2O_3 having a mean particle diameter of approximately 91 nm, about one order of magnitude *less* than the minimum diameter required [38,40] for flocculation by a static magnetic field of 0.1 T. To insure that our colloidal preparation was stable, we tested our starting preparation at a pH of 4.2 using different concentrations of supporting electrolyte and determined that the critical coagulation concentration of our preparation was about 50–60 mM KCl [34].

An inert, recirculating test loop was constructed entirely out of Tygon and glass to avoid any possibility of contamination of the test suspension with corrosion products [34]. The apparatus was closed to the atmosphere, and the temperature, conductivity, and pH could be continuously monitored and controlled. A peristaltic pump was used to circulate the suspension around the treatment loop at a linear velocity of 0.83 m s^{-1}. Two sets of permanent magnets with a magnetic induction of about 0.15 T were arranged *externally* around the Tygon tubing so that the magnets never came into direct physical contact with the suspensions. The direction of the field was orthogonal to the direction of fluid flow.

Three treatment methods were employed: (1) dynamic magnetic treatment during which the sol was circulated through the test loop in the presence of the field, (2) static magnetic treatment during which the sol was allowed to stand in the field without circulation, and (3) circulation through the test loop in the absence of the applied field. In all three methods, the sol was first circulated in the test loop for 30 min to stabilize the temperature, conductance,

pH and aggregation state of the suspension under flow conditions.

During each test, the particle size of our sol was measured at the beginning (after 30 min of circulation) and then at regular intervals over a 120-min treatment period. Initially, the aggregation state was monitored using the 90° scattered light intensity of 575 nm incident radiation, after first establishing the relationship between these two parameters using addition of electrolyte to induce a known change in aggregation state [34]. As shown in Fig. 6, these results were later confirmed by particle size distribution measurements [41,42] using photon correlation spectroscopy [43].

The results of these experiments showed that at a supporting electrolyte concentration of 30 mM KCl (just below the critical coagulation concentration), hematite sol underwent aggregation when exposed to the magnetic field under flowing conditions [34,41,42]. By comparison, neither continued circulation in the absence of the magnetic field or exposure to the magnetic field in the absence of fluid flow produced a comparable change in the aggregation state [34,41,42]. Since both the magnetic field and fluid flow were required to produce aggregation, the effect is most properly termed magneto-hydrodynamic aggregation [42].

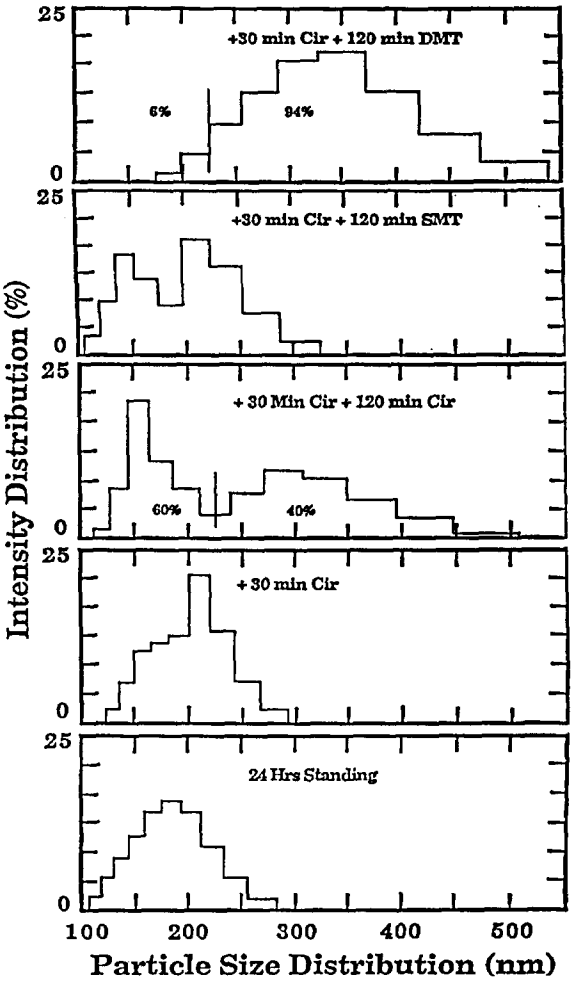

Fig. 6. Particle size distribution in hematite sol (in 30 mM KCl) following different forms of treatment. From bottom to top: (1) after standing 24 h in 30 mM KCl; (2) after 30 min circulation in the absence of the magnetic field; (3) after 150 min circulation in the absence of the magnetic field; (4) after 30 min circulation in the absence of the magnetic field and another 120 min standing in a 2.0T magnetic field (no further circulation); (5) after 30 min circulation in the absence of the magnetic field and another 120 min circulation in the presence of a 2.0 T magnetic field. Percentages represent the quantity of sol having a diameter above or below 225 nm [42].

4.2. Colloidal latex and cholesterol

Since hematite is weakly ferromagnetic, it might still be argued that the aggregation effect we observed resulted from magnetization of the solid phase. To determine whether magneto-hydrodynamic aggregation was a general effect, independent of the magnetic properties of the solid phase, we next investigated suspensions of cholesterol and standard polystyrene latex beads [42,44,45], both of which are diamagnetic solids.

For these experiments, a variable strength electromagnet, capable of producing magnetic inductions of up to 2.0 T, was substituted for the 0.15 T permanent magnets used previously [34,41]. Cholesterol suspensions were prepared by a change of solvent method [46,47] and stabilized by the addition of the bile salt, sodium taurodeoxycholate monohydrate. We found that

the magnetohydrodynamic aggregation effect in cholesterol could be observed most readily in 50 mM NaCl supporting electrolyte, a value which produced a suspension on the verge of instability due to double-layer compression. Standard polybead polystyrene microspheres with a particle size of 150±16 nm were also studied. Stock solutions of the polystyrene latex were diluted in 200 mM NaCl to produce a colloidal suspension that was also on the verge of instability.

Magnetic treatment was performed under dynamic (flowing) and static (no flow) conditions. In some experiments, the samples were exposed to the magnetic field at the very beginning of the experiment, while in other experiments, samples were circulated for 30 min prior to the application of the magnetic field. For the cholesterol suspensions, each type of treatment was conducted at three different values of magnetic induction—0.15 T, 1.0 T, and 2.0 T. Control experiments were performed by circulation of the suspension without the application of the magnetic field. Each experiment was repeated three times and blanks were repeated six times. Particle size measurements were obtained by photon correlation spectroscopy.

Results [45] obtained from multiple runs using cholesterol and latex showed that flowing these suspensions through an orthogonally applied magnetic field resulted in increased aggregation compared with exposure to the magnetic field in the absence of flow or flow in the absence of the magnetic field (Figs. 7–9). It was also interesting that in the case of cholesterol, aggregation under dynamic treatment conditions was not directly proportional to magnetic field strength, but appeared greater at 0.15 T and 1.0 T than at 2.0 T (Fig. 8). The 30 min of circulation prior to initiation of magnetic treatment was not a significant factor in overall aggregation rate (other than delaying the onset of significant aggregation by 30 min), and once the magnetic field was switched on, periods of aggregation were regularly followed by periods of apparent deaggregation or particle reorganization (Fig. 8). Since the difference between treatment methods was most pronounced for suspensions with electrolyte concentrations near the critical coagulation concentration, we concluded that the magnetohydrodynamic aggregation effect is relatively small, and continued recirculation is needed to show a significant particle size difference between flow in the presence of the magnetic field and flow in the absence of the field [45].

4.3. *Interpretation of colloid studies*

During flocculation, aggregates form as a result of collisions between particulates [48,49]. If these collisions result from Brownian motion, the process is known as perikinetic flocculation [50]. If these collisions are due to differential particulate velocities (velocity gradients) produced by mechanical means, the process is known as orthokinetic or shear flocculation [50]. Since perikinetic flocculation is generally slow, orthokinetic processes, which depend on the hydrodynamics of the system, are frequently the controlling factor [51,52].

Because the colloidal aggregation/deaggregation we have observed under dynamic magnetic treatment appears to be independent of the magnetic susceptibility of the solid phase and is not directly proportional to field strength, the effect cannot result from direct interaction of the field with the colloidal particles. Instead, we attribute [44,45] the effect primarily to an orthokinetic phenomenon [51–56] that arises from magnetohydrodynamically produced alterations in the fluid flow conditions and produces differential velocity gradients for the colloidal particles. Support for this hypothesis can be found in the theory behind flocculation [48,49] and in the well-established, although not widely known, influence of magnetic fields on the hydrodynamic properties of conducting fluids [23,26].

In 1937, Hartmann and Lazarus [57] showed that if the magnetic field is strong enough, the

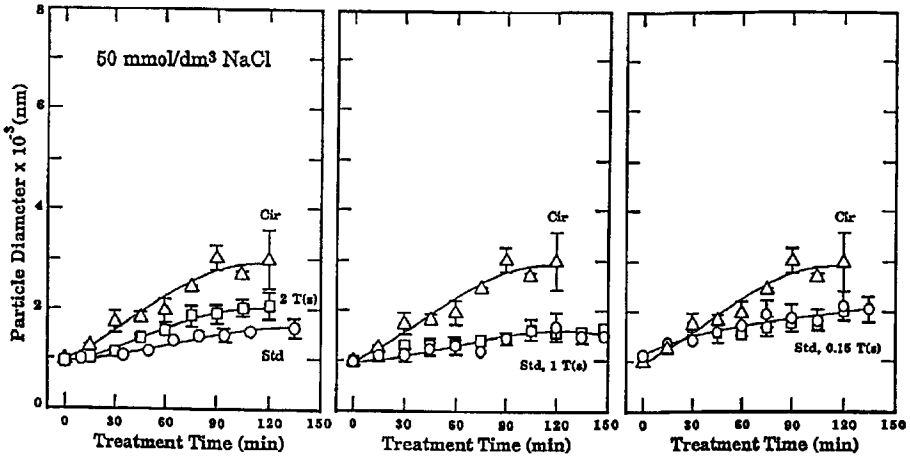

Fig. 7. Effect of (□) static magnetic treatment on colloidal cholesterol/NaTDC (standing in the presence of the field) compared with (Δ) circulation in the absence of the field and (○) standing in the absence of the field. Electrolyte concentration, 50 mM NaCl. Flux density from left to right: 2.0 T, 1.0 T, and 0.15 T. Error bars represent ± 1 SD of the mean. Magnetic field applied at treatment time 0 (no prior circulation). Particle size at 0 treatment time represents the aggregation state resulting from 1 h standing following addition of electrolyte. (Reprinted from [45].)

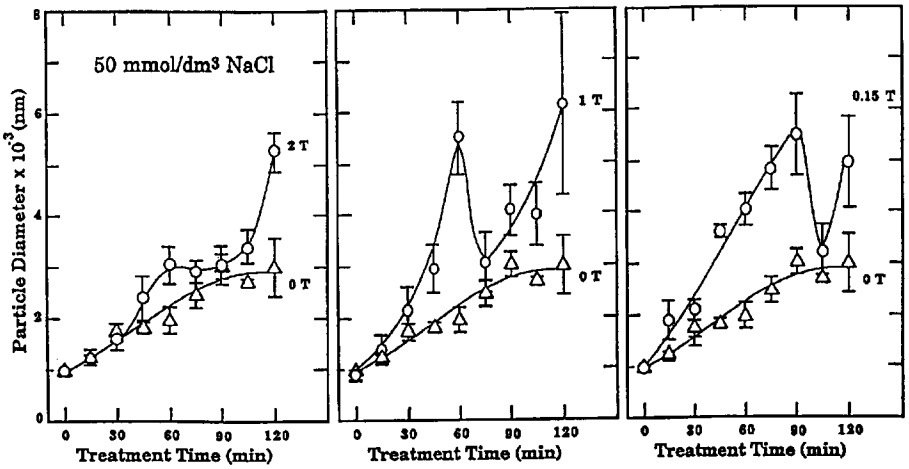

Fig. 8. Effect of (○) dynamic magnetic treatment on colloidal cholesterol/NaTDC (circulation in the presence of the field) compared with (Δ) circulation in the absence of the field. Electrolyte concentration, 50 mM NaCl. Flux density from left to right: 2.0 T, 1.0 T, and 0.15 T. Error bars represent ± 1 SD of the mean. Magnetic field applied at treatment time 0 (no prior circulation). Particle size at 0 treatment time represents the aggregation state resulting from 1 h standing following addition of electrolyte. (Reprinted from [45].)

velocity profile of a flowing fluid will be flattened in the center of the conduit, and a distinct boundary layer will develop adjacent to the conduit wall. They also observed [57] that the application of even a relatively weak magnetic field orthogonal to the direction of a weakly turbulent flow will decrease the pressure gradient required for a given flow rate, the primary effect being attributed to a reduction of electrical eddies in the fluid. The magnetic field can also affect the

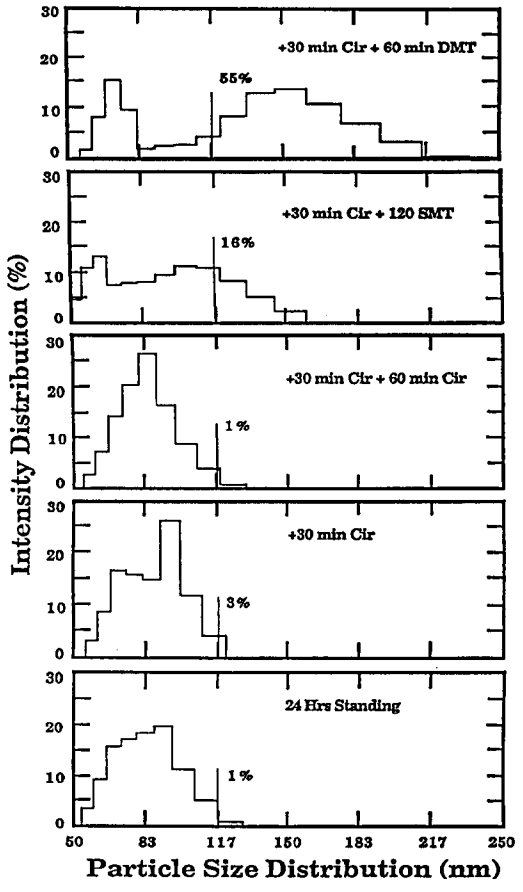

Fig. 9. Particle size distribution in polystyrene latex suspensions (in 200 mM NaCl) following different forms of treatment. From bottom to top: (1) after standing 24 h in 200 mM NaCl; (2) after standing 24 h in 200 mM NaCl followed by 30 min circulation in the absence of the magnetic field; (3) after standing 24 h in 200 mM NaCl followed by 90 min circulation in the absence of the magnetic field; (4) after standing 24 h in 200 mM NaCl followed by 30 min circulation in the absence of the magnetic field and another 120 min standing in a 2.0 T magnetic field (no further circulation); (5) after standing 24 h in 200 mM NaCl followed by 30 min circulation in the absence of the magnetic field and another 60 min circulation in the presence of a 2.0 T magnetic field. Percentages indicate the quantity of latex particles having a diameter above 117 nm [42].

vorticity of the fluid flow, increasing or decreasing the state of rotation of the fluid [23,58,59] depending on experimental conditions. Such magnetically induced velocity gradients can be shown to promote flocculation as well as floc breakup [48], as we observed in the case of cholesterol (Fig. 8).

For the conditions used in our colloid experiments [42,44,45], we calculated [45] the change in velocity profile across the tube for a magnetic induction of 1.0 T and concluded that the field was too small to develop a distinct boundary layer near the walls of the tube. However, even small changes in the velocity profile across the tube (calculated as a 1.3% reduction) could alter the shear forces experienced by the suspended particles. As indicated by our experiments, the observed magnetohydrodynamic aggregation/deaggregation effects are, in fact, rather small and can only be detected for colloidal suspensions on the verge of instability.

5. Relationship of this research to magnetic water treatment

The following discussion strictly pertains to magnetic treatment devices in which the magnetic field is orthogonal to the fluid flow. On the basis of over ten years of research in this field, we have reached the following conclusions:

1. If placed in direct contact with the flowing fluid, MTDs can release a variety of corrosion products during operation as a result of magnetohydrodynamically generated stray currents. This may be an intentional design feature of the device or simply an artifact of operation.

2. Some commercial MTDs have a zinc or aluminum sacrificial anode to reduce corrosion of the housing. These units should release either zinc or aluminum ions that may subsequently hydrolyze to form zinc or aluminum hydroxides. In the absence of a sacrificial anode, an iron colloid can be generated as a result of enhanced corrosion of the MTD housing. Any of these corrosion products could affect the precipitation of scale-forming minerals.

3. In cathodic regions of the corroding devices, high surface pH conditions exist because of the release of hydroxide ions from the reduction of water or oxygen. This region of high pH could induce localized formation of seed crystals of certain scale-forming minerals such as $CaCO_3$.

4. Our Phase II test protocol shows that MTDs can produce a statistically significant reduction in calcium carbonate scaling for certain artificial waters. The observed reduction (22%) is not much larger than the reduction obtained using the unmagnetized MTD (17%). There was no statistically significant reduction in either magnesium or calcium carbonate scale when Brazos River water was used. This may mean that a number of factors influence MTD efficacy, and we do not yet know what all these factors are. The results also suggest that the presence of the magnetic field may have less of an influence on scale reduction than the changes in fluid flow patterns induced by the presence of obstructions inside the pipe.

5. We suspect that under suitable conditions, MTDs can promote flocculation of metastable colloids. The flocculation phenomenon appears to be quite general and is observed with diamagnetic as well as weakly ferromagnetic materials. We believe this flocculation is orthokinetic in nature, resulting from the magnetohydrodynamic generation of shear forces in the fluid flow. The voltage measurements in Phase I of our studies confirm the generation of an electric field by magnetohydrodynamic processes. It is the generation of this electric field that leads to the production of electrical eddy currents and alterations in fluid flow conditions.

6. From Phase III studies, we must conclude that magnetohydrodynamic aggregation is a small effect because we required colloidal suspensions on the verge of instability and recirculation of the colloids through the field for periods of 30 min or more before the phenomenon could be reproducibly demonstrated.

7. The energy required to alter fluid flow conditions is supplied by the pump, which must work harder to force the conducting fluid through the magnetic field. Thus, the MTD behaves like an electromagnetic brake [24]. To produce the colloidal aggregation observed in our studies required a magnetic field of appropriate strength, a fluid flow orthogonal to the magnetic field, and an adequate solution conductivity. We could increase magnetohydrodynamic forces by increasing solution conductivity [23], but once the critical coagulation concentration of our colloids is exceeded, the colloids will aggregate spontaneously due to double layer compression [34,45].

8. If magnetohydrodynamic forces become large enough, the shear forces generated in the fluid may become large enough to promote floc break-up (Fig. 8). This means that MTDs might be operated in either an aggregating or deaggregating mode, depending on experimental conditions.

If the results enumerated above have any bearing on the scale-reducing properties claimed for MTDs, we would predict that the following conditions will be necessary for successful operation of the device.

1. The MTD will require a sufficiently fast, continuous flow of fluid. If magnetohydrodynamic forces are responsible for the action of the device, continuous fluid flow is required to generate these forces.

2. The magnetic field must be of sufficient strength and oriented 90° relative to the direction of fluid flow. Only when the conducting fluid cuts the magnetic lines of force will magnetohydrodynamic forces of the type described here be generated.

3. By Ohm's Law, the magnitude of the current produced by fluid flow through the field is proportional to σ, the solution conductivity (Fig. 1). Thus, the treated solution must have sufficient conductivity to produce a current.

If these conditions are required for proper MTD operation, then it is hardly surprising that

tests employing hot water heaters have been generally negative [8]. Fluid flow in such systems is typically slow and intermittent. Other types of once-through testing should also produce negative results [9]. On the other hand, it should not be surprising that two of the most recent field successes reported for these devices [12–14] involved constantly *recirculating* cooling towers where sufficient linear fluid flow velocity was maintained continually, the solution was repeatedly recirculated through the MTD, and the solution conductivity was relatively high and increased with increasing cycles of concentration.

In both cooling tower applications [12–14], the MTD was installed on the discharge side of the pump, just prior to the heat exchanger. Some manufacturers indicate that their MTD must be installed in close proximity to the heat exchanger, further specifying that no elbows or bends be present in the intervening pipe. Again, if magnetohydrodynamic factors are involved, the electrical eddy currents and forces produced in the fluid by the MTD are not likely to persist into the heat exchanger if the MTD is too far away or if bends and elbows are present in the intervening pipe.

If MTDs are capable of preventing scale deposition, as their manufacturers and certain reports claim [12–14], the effect may be due to a combination of factors. Aside from the possible production of nucleation centers as a result of electrolysis—which, although possible, has yet to be experimentally demonstrated—the devices may simply function by creating turbulence. It is known, for example, that scaling minerals generally precipitate more readily in turbulent areas [2], and mechanical turbulence promoters are sometimes used to improve heat transfer and prevent deposit formation in certain types of evaporators [2].

In the case of the MTDs used in Phase I of this study, the production of turbulence was a readily observable characteristic of their operation. These devices seemed, in fact, to be designed to promote turbulence in the fluid flow, not only by magnetohydrodynamic means, but also by mechanical means. The design of the Phase I commercial unit included pole pieces that not only focused the magnetic field, but also reduced the cross-sectional area of the conduit by a factor of 4 in the vicinity of the pole piece gap. This produced turbulence in the downstream fluid. Also, behind each pole piece, the presence of fluid flow eddies could be visually observed in regions of the interior shielded by the pole pieces. Finally, fluid flowing through the MTD experiences two successive encounters with the pole pieces, causing the fluid to emerge as a highly turbulent jet at the downstream end of the device. Under appropriate conditions, this turbulence might generate seed crystals which could be swept into the heat exchanger and act as nucleation centers for precipitation of scale-forming minerals in the bulk of the solution. Turbulence should also promote degassing of the solution, and loss of dissolved CO_2 will affect the pH and carbonate concentration. Whether or not this could affect scaling will depend on water conditions.

Our Phase II studies also support the hypothesis that turbulence is an important factor in scale reduction. In these experiments, the unmagnetized MTD was found to produce nearly the same amount of scale reduction (17%) as the magnetized unit (22%). Neglecting any effects resulting from corrosion products, these results might initially be taken to suggest that turbulence due to the presence of the pole pieces produces 17% of the total scale reduction, while magnetohydrodynamic forces accounts for only another 5%.

6. Future directions

In our research, we have attempted to identify *measurable* effects that occur reproducibly as a result of MTD operation and that are based on *well-established scientific principles*. We have seen that laboratory measurements do show that

magnetohydrodynamic effects can occur with aqueous conducting solutions that are forced to flow through an orthogonally applied magnetic field. The effects are small. However, if magnetic water treatment produced large, obvious effects, there would be no controversy surrounding the mode of action or its effectiveness.

At this point, we have identified several effects that are likely to occur in the type of MTD studied in this research. However, we cannot yet prove that these effects are directly linked to the claims of scale reduction, nor do we know whether other effects, yet to be uncovered, are also occurring. However, because the effects uncovered, to date, can be reasonably related to scale deposition, we believe that magnetic water treatment should not be dismissed out of hand. This does not mean that every magnetic treatment device currently on the market actually works or that those that do work are effective under every possible scaling situation. Even if magnetic treatment can be shown to reduce scaling conditions when used properly, the question remains as to whether the technology is robust enough in comparison with chemical means of scale prevention. If magnetic treatment devices are effective, and we are finally able to understand how they operate, the best strategy could be a combination of magnetic and chemical treatment.

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